The spectroscope is pushed up partly into the tube of the refractor so that the slit coincides with the corrected focus.

In this arrangement many advantages are gained, notably the following :—

(1) An improved colour correction results.

(2) Strength is gained in the attachment of the spectroscope to the eye-end.

(3) Space is economised, for the spectroscope is 18 inches nearer to the object-glass of the refractor.

(4) The whole spectroscope, being attached to a strong framework which is clamped to the focussing tube of the refractor, can be moved bodily in and out (for the purpose of focussing the star on the slit) without altering the adjustment of the parts of the spectroscope.

(5) The convergency of the cone of rays from the object-glass of the refractor can be arranged to have a very convenient value ; the convergency for the uncorrected object-glass is about 1 in 14·0, and with the correcting lens it becomes 1 in 10.

(6) As a consequence of the altered convergency, the requisite resolving power can be attained in the single prism with shorter collimator.

The one drawback that I realise at present is that, since the relation between the purity P, the resolving power R, and slit width s, and the ratio ψ of aperture to focal length, is

$$P = \frac{\lambda}{s\psi + \lambda} R,$$

it is clear that the slit-width must, for a given purity and resolving power, vary inversely as ψ. This I regard as a great disadvantage ; but it has seemed to me that there was a balance of advantage in favour of the lens.

I was at first inclined to think that the inaccessibility of the slit was an insuperable objection. But the adoption of Huggins' admirable plan of a reflecting slit-plate has got over all the anticipated difficulties.

Having thus briefly indicated the general method adopted, I proceed to describe some of the details.

The Eye-end or Breech-piece of the 25-inch Refractor.

No doubt many of the conveniences of the adopted method of attaching the spectroscope depend on the arrangement of the eye-end of this special refractor. The sturdy massiveness of Cooke's work has formed a splendid foundation, to which the spectroscope has been fitted.

The steel tube of the refractor is cigar-shaped, wider in the middle than at the ends. At the eye-end the steel tube has a diameter of 21 inches, and to it is fitted a strong iron casting

which contracts the opening with a rapid taper down to $8\frac{1}{2}$ inches, and forms a strengthening piece with a turned flange. Into the opening thus left is fitted a massive breech-piece (an arrangement of draw tubes, position circle and focussing mechanism), ending in a flange with a kind of bayonet joint, to which the various adapters for eye-pieces, micrometer solar eye-pieces, &c., can be fitted. All apparatus fitted to the bayonet joint can be rotated in connection with the position circle, and can be racked in and out by means of the focussing screw through a range of 4 inches. The breech-piece weighs about $1\frac{1}{2}$ cwt., and its weight gives an idea of its strength. It is to this bayonet-joint flange, the aperture of which is $5\frac{3}{4}$ inches in diameter, that the spectroscopic appliances are attached. The plane of the flange, when racked in as far as the focussing screw will take it, is about $7\frac{3}{4}$ inches nearer to the object-glass than the uncorrected focal plane.

The Correcting Lens.

A simple convexo-concave lens of aperture 5 inches, and of focal length 154 inches for light of wave-length 5890–6 λ is set in the convergent beam of rays coming from the object-glass of the refractor at a distance of about 62 inches from the focus, or, as I shall now call it, the uncorrected focus. The corrected focus is about 18 inches nearer to the object-glass.

The lens is mounted at the end of a brass tube, and the other end of the tube is provided with a heavy flange. The tube is pushed, lens first, into the refractor, and the flange is clamped into the bayonet joint at the end of the breech-piece.

The position of the lens can be altered by the focussing screw, but when the lens is pushed in as far as it will go, then the new, or corrected, focus is inside the tube which holds the correcting lens about 11 inches from the new flange on the breech-piece.

It is unnecessary to go into details concerning such a lens, inasmuch as Keeler has recently published (*Astroph. Jour.* 1895, i. p. 101) a note on work which is in great measure identical with that which I undertook in considering the possibility of getting a satisfactory improvement of the colour curve with a simple lens. Keeler has rejected the solution "for the general case of large telescopes," on the ground that the alteration which the use of such a lens would produce in the aperture of the convergent beam (*i.e.* the ratio of the diameter of the cross section of the convergent beam to the distance of the cross section considered from the focal plane) is excessive. In the case he considers the ratio is altered from 1 : 19 to 1 : 5, and this would involve the use of a collimator of such unusual proportions as to be impracticable.

But the question is—is it possible to produce a considerable change in the colour correction without excessive change in the ratio referred to? Elementary calculations, similar to those published by Keeler, showed that it was worth while to have a lens

made, and experimental determinations of the corrected separation of the foci for different colours for the actual correcting lens used have convinced me that the improvement is considerable. It is clear that if the separation of the foci were reduced only in the same proportion as the convergency ratio, no advantage would be gained; when one of two colours was in focus on the slit the circle of aberration for the other colour would be just as great as without the correcting lens.

My lens is arranged to give a convergent beam, with ratio 1 : 10, whilst the uncorrected object-glass has a ratio 1 : 14·0. Under these circumstances the collimator of the spectroscope is of very convenient dimensions—namely, 2 inches aperture and 20 inches focal length. A comparison of the diameter of the circles of aberration on the slit, first for the uncorrected object-glass and second for the corrected object-glass, shows clearly the advantage gained. If the light focussed on the slit in each case is light of wave-length 4860 λ (H_β), the circles of aberration deduced from actual measurement for H_γ and H_δ have diameters as follows:—

	Uncorrected O.G. mm.	With Correcting Lens. mm.
H_β	0·00	0·00
H_γ	0·81	0·36
H_δ	1·94	0·97

Photographs of star spectra are satisfactorily uniform from λ 5896 (D) to λ 4470. I refer here to uniformity of density; in another place I give suggestions as to a cause of unsatisfactory definition at the ends.

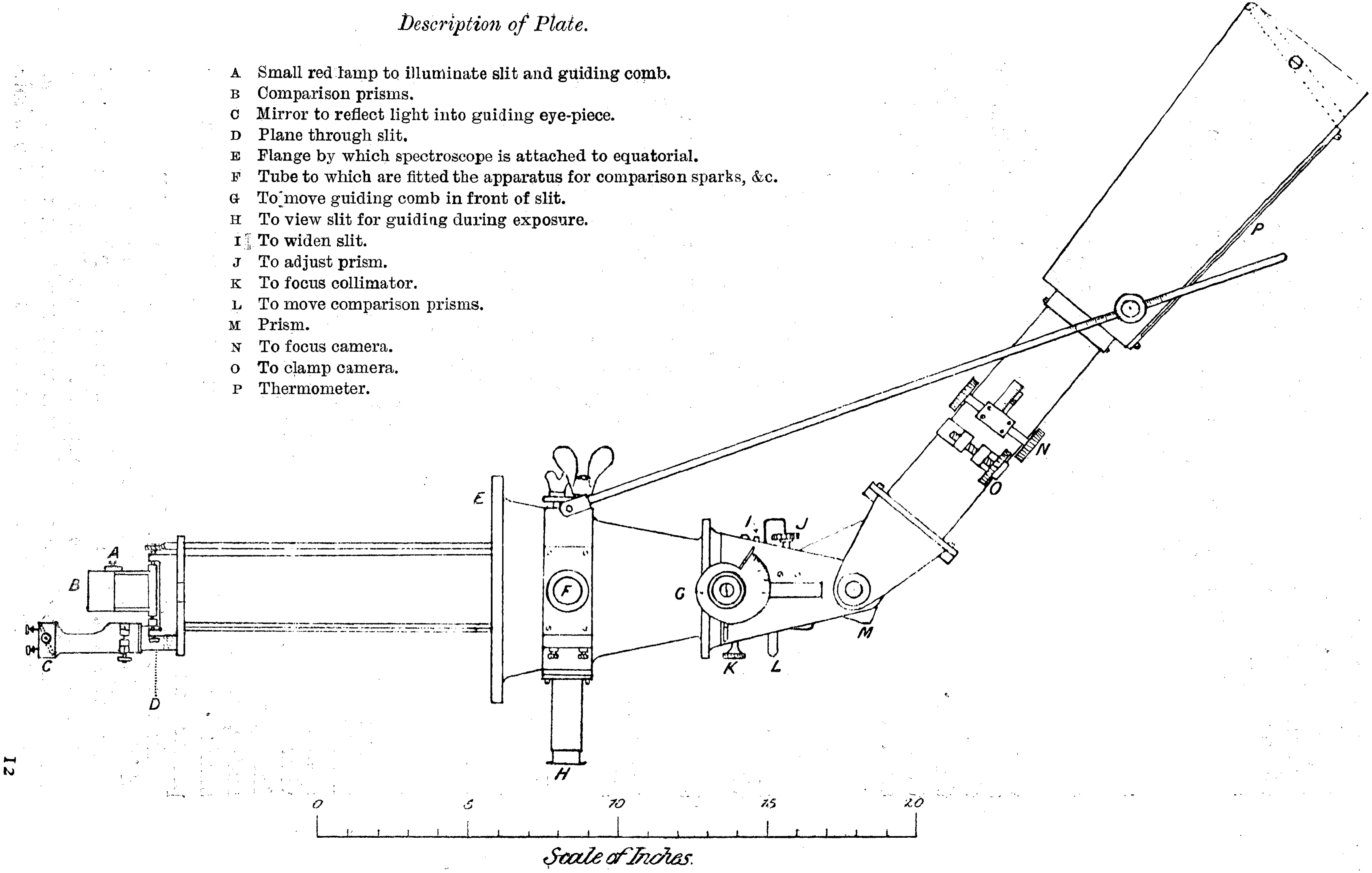
The following point with respect to the focussing of the star on the slit may be noted. The distance between the slit and the correcting lens is fixed when the collimator is clamped in the framework. The focussing is accomplished by moving both spectroscope and correcting lens simultaneously in or out by means of the focussing screw. The distances between the correcting lens and the conjugate foci are so related that the movement of the lens and spectroscope through any given small distance produces a movement of the star-image through nearly exactly one-half that distance with respect to the slit. The focussing, which is of great importance, can thus be done with great accuracy.

The Framework of the Spectroscope.

The frame of the spectroscope consists of a heavy hollow conical casting of gun-metal with a flange at each end. The larger flange is that by which the whole spectroscope is clamped to the equatorial by means of four large thumb-screws; to the smaller flange is attached a strong ribbed aluminium casting, carrying the pivots about which the whole camera can turn, and between which the prism is mounted.

Description of Plate.

A Small red lamp to illuminate slit and guiding comb.
B Comparison prisms.
C Mirror to reflect light into guiding eye-piece.
D Plane through slit.
E Flange by which spectroscope is attached to equatorial.
F Tube to which are fitted the apparatus for comparison sparks, &c.
G To move guiding comb in front of slit.
H To view slit for guiding during exposure.
I To widen slit.
J To adjust prism.
K To focus collimator.
L To move comparison prisms.
M Prism.
N To focus camera.
O To clamp camera.
P Thermometer.



The collimator is held in the conical casting, the lens projecting through a hole in the smaller flange and the aluminium casting, and the plane of the slit being about 11 inches from the plane of the larger flange. The whole collimator is arranged to slide through a small range (about $\frac{1}{2}$ inch) so that the distance between the slit and the flange may be adjusted. The final focussing of the star-image on the slit is accomplished by moving the whole spectroscope in or out by means of the large focussing screw on the breech-piece.

About midway between the flanges on the conical casting the casting is thickened, so that a cylindrical ring is formed which facilitates the attachment of several accessory arrangements: (i) a telescope and reflectors to enable the observer to view the slit as from in front; (ii) condensing lenses and reflectors to throw an image of a spark or tube for comparison spectra upon the slit; (iii) a clamping-screw to hold the stay-rods by which the camera is prevented from turning about the pivots; and several other small things which it is not necessary to specify.

The Collimator and Slit and Guiding Comb.

The collimator has a focal length of $20\frac{1}{2}$ inches (520 mm.) and an aperture of $2\frac{1}{8}$ inches (54 mm.), the object-glass being a visual achromatic.

The stout collimator tube is made so that it can slide through a small range in the frame of the spectroscope; and when the tube is clamped in position the object-glass can be moved relatively to the slit by means of a rack and pinion. A scale is attached by which the focus-reading can be read off.

The slit is arranged after the admirable device of Dr. Huggins. The jaws are made of speculum-metal, and the exposed faces are highly polished, so as to form a single plane surface, which is inclined at a small angle to the axis of collimation. Great care was taken to work the sharp edges in a proper manner.

When the image of a star is thrown upon the slit, some portion of the light passes through the slit; the rest is reflected by the polished faces of the jaws on to a small mirror, fixed in front of the slit and displaced slightly to one side, so as not to interfere with the incident pencil. The mirror, together with a system of lenses and a reflecting prism, enables the observer to view the slit: he looks into an eyepiece attached to the conical framework in a direction perpendicular to the axis of the collimator, and sees the slit and any images (whether of star or of spark for comparison spectra) that may fall upon it from the proper quarter.

In front of the slit is set a small movable guiding comb, which enables the observer to set the star-image on any required part of the slit. The teeth of the comb cover certain parts of the slit, and leave the rest exposed. By a suitable mechanism the comb can be either moved by a very small amount up and down the slit, or altogether withdrawn so as to expose the whole

slit. By making the teeth of the comb twice as wide as the gap between them, it is arranged that three spectra can be set side by side—*e.g.* a star spectrum taken between two spark spectra, one of which is taken before, the other after, the star spectrum. In this mode of procedure any changes of adjustment that may have arisen during the exposure for the star spectrum, in consequence of change of temperature or in the position of the spectroscope, may be at once detected in the photograph.

The beautiful device of Dr. Huggins's reflecting slit is only open to one objection, so far as I am aware. Let us suppose it is desired to investigate a spectrum near Hγ. In this case it is necessary to focus Hγ-light on the slit and to keep it on the slit. It is difficult to do this, in consequence of the chromatic aberration of the equatorial; but the following device has proved efficacious. From time to time the star is observed on the slit with a small direct-vision compound prism held between the eye and the guiding eyepiece in such a way that the length of the spectrum is parallel to the length of the slit. The slit then appears as a fine dark line running along the length of the spectrum, which is narrow at the part or parts focussed on the slit, and has at any other part a width determined by the diameter of the circle of aberration of the light corresponding to that part. The star is then moved until the narrow parts of the spectrum fall on the slit. The prism is then removed, and the position of the slit with respect to the slightly blurred star-image is noted, so that guiding can be continued with only occasional recourse to the prism.

In such work as the investigation of the spectrum of special parts of a planet or a nebula the reflecting slit is invaluable.

The Prism.

The prism at present used is a white dense flint prism of 60°. The height is $2\frac{1}{8}$ inches (54 mm.), and the length of the side of the triangular section is $3\frac{3}{8}$ inches (86 mm.).

The resolving power	for	λ 5896	is about	7600;	$\delta\lambda =$	·8	tenth-metre.
"	"	λ 5000	"	12000	"	·4	"
"	"	λ 4250	"	22000	"	·2	"

The edges and angles of the prism are all ground blunt, and the prism is partly encased in aluminium sheet, which is bent so as to cover the triangular faces (the "top" and the "bottom") of the prism and also the ground rectangular base, but so as to leave the two polished rectangular faces free. Two small gun-metal bosses are fitted to the aluminium case, one fixed on the bottom, the other being adjustable within small limits on the top of the prism. Fine centre-holes are drilled through the bosses and the line joining them is made parallel to the refracting edge, the final adjustment being made by moving the adjustable boss by four crews. Two screws with fine conical points pass through the pivots about which the camera turns, and the prism is held in its

case between these conical points or male centres, which are screwed through the pivots into the female centres in the bosses. The prism thus held is free to turn about an axis through the male centres. An arm projecting from the top of the prism-case is pressed by a spring against the end of a screw which is fixed to the frame-work, and a slow motion for adjusting the prism for minimum deviation is thus provided.

This mode of holding the prism has proved very satisfactory. The slit at the end of the collimator is adjusted to parallelism with the line joining the male centres, and this ensures parallelism with the refracting edge of the prism, provided that the line through the female centres has been adjusted.

The Camera.

The camera is made of a brass tube about 10 inches long joined rigidly to a tapered box of rectangular section, made of aluminium sheet and having a length of 10 inches. The total length of the camera is thus 20 inches.

The following object-glasses can be used in it, each having an aperture of $2\frac{1}{8}$ inches (54 mm.).

(1) A visual achromatic object-glass of focal length 20 inches (508 mm.).

(2) A plano-convex quartz lens of focal length 20 inches. The use of an uncorrected lens of this kind is convenient when flat photographic plates are used. With two similar achromatic lenses in the collimator and camera, the result of the over-correction of both lenses is to give a spectrum which cannot be in focus over a considerable range unless a curved plate or film is used. If an uncorrected lens is used in the camera in connection with an over-corrected lens in the collimator, the spectrum is flatter. Whether it is better to use glass or quartz for the simple lens depends upon the character of the colour correction of the achromatic object-glass used in the collimator.

In laboratory work I have used a spectacle lens of 36 inches focal length in connection with an over-corrected "achromatic" collimating lens and have got admirable spectra from the D lines to the λ 3800 photographed in sharp focus on a single flat plate without tilting the plate. I have not seen the method described, but it is so simple that it is most probably known.

(3) A telephoto-combination, designed and used in such a way as to have an equivalent focal length of about 40 inches (1016 mm.) though the extreme *actual* length of the camera is the same as with the other object-glasses, namely 20 inches (508 mm.).

The use of this optical device was only decided upon after numerous experiments, and I take this opportunity of expressing my obligations to Messrs. Dallmeyer for their kindness in letting me try some of their combinations before having a pair made for use in this spectroscope.

If justification of the use of this method be required, it may be based on the following considerations. The prism used is of such dimensions that the resolving power is considerably higher than that which a photographic film with its markedly granular structure can ever do justice to. The theoretical resolving power of my prism is a little more than one of Professor Vogel's compound prisms. Professor Vogel * has expressed the opinion that the performance of his prisms would have warranted a large increase in the focal length of the camera, but such an increase would have increased the linear dispersion so much that only the brightest stars could have been observed with the Potsdam Refractor. The question therefore presents itself: Is it better to use two prisms and a short camera or one prism and a long camera? The only considerations which bear on the point are practical, and it appears to me that the coarseness of granularity of available photographic plates is the most important factor; for it would seem useless to employ an optical resolving power greater (except by an arbitrary amount for margin) than the defining power of the photographic plate. If the necessary resolving power can be attained with a single prism of manageable dimensions, the balance is in favour of the single prism, for it involves a smaller loss of light.

Scales on the Instrument.

The importance and convenience of having scales on the instrument, by which every adjustment of every adjustable part can be recorded, cannot be over-estimated.

The following scale readings are taken for each exposure made on the telescope.

1. Position angle of the slit, usually 90°.

2. Focus reading, localising the position of the slit amongst the coloured images of the star formed on the collimation axis of the equatorial.

3 and 4. Focus readings of the collimator and camera.

5. Inclination of the photographic plate.

6. The inclination of the axes of camera and collimator.

7. Width of the slit.

8. Temperature recorded by a thermometer attached to the camera.

9. The part of the slit exposed at different times during any exposure.

There is still needed a tenth scale to record the position of the prism.

Some Numerical Details.

The weight of the spectroscope is 26 lbs.; the weight of the flanged tube which holds the correcting lens, 13 lbs.

Assuming, for the sake of definiteness, that determinations of radial velocity would be made by measurements near H_γ, the

* *Publn. d. Astroph. Obs. zu Potsdam*, 1892, vol. vii. pp. 20 and 21.

following details give a more precise idea of the conditions under which such measurements could be made with the spectroscope described in this note.

The spectrum near H_γ has a linear dispersion 1 mm. to 21 λ (tenth-metres). Taking the minimum measurable quantity to be $0^{mm}\cdot001$, this would correspond to $0^{\lambda}\cdot021$, or to 1·5 km./sec., or 0·9 mile/sec. (In Vogel's classical researches, $0^{mm}\cdot001$ corresponded to $0^{\lambda}\cdot013$, or 0·9 km./sec., or 0·6 mile/sec.)

The diameter of the first diffraction ring in the corrected image formed by the object-glass of the equatorial and the correcting lens is, for H_γ, $0^{mm}\cdot011$.

Near H_γ, since the resolving power of the prism is 22000, the purity of the spectrum can never be advantageously greater than 7000. The following table shows the relation between the purity and the slit-width.

Purity.	Slit-width. mm.	Spectrum pure enough to separate two lines for which $\delta\lambda$ is
7300	0·0086	0·60 tenth metre
5500	0·025	0·79 ,,
4340	0·035	1·00 ,,

Actual measurement shows that the distance between the centres of neighbouring grains on a photographic film of average goodness lies between $0^{mm}\cdot01$ and $0^{mm}\cdot035$, and a fair average value seems to be $0^{mm}\cdot02$ or $0^{mm}\cdot025$. With the telephoto camera the image of the slit is twice as wide as the actual slit. It is thus seen that, with a slit-width $0^{mm}\cdot025$, the image on the plate involves two grains in its width.

It will be noted that the width of slit is, in the case just dealt with, twice as wide as the diameter of the first diffraction-ring in the star image. It would seem to me to be convenient to introduce the term "tremor-disc." The name more or less explains itself: it is easiest to state what it is intended to convey by reference to a photograph of a star taken with a long exposure. The star-image moves about on the plate in consequence of atmospheric tremor, and produces its effect at each spot on which it rests; the developed image is strongest where the star has most frequently rested; the distribution of density is probably symmetrical about the mean position of the star, and the intensity at different points along a diameter of the resulting tremor-disc is probably fairly well represented by a "law of errors" curve. Apart from the photograph, which shows the summation of the effects, the tremor-disc may be conceived as existing in time, so to speak; and the effect produced in a slit-spectroscope depends on the relation between a certain area of tremor-disc and the area of the slit illumined by it. The tremor-disc is of greater importance, so far as the design of a stellar spectroscope is concerned, than the diffraction-disc, which has generally been considered.

The tremor-disc at Cambridge frequently has a diameter of

8″ or even 10″; and on average nights it is probably fully 5″. Assuming that the bright central part is 3″, this would be a disc whose linear diameter is 0·09 mm., and whose area is three times as great as the illumined part of the widest slit which it is thought advisable to use.

The instrument has been named the Bruce Spectroscope, since it is one of the numerous outcomes of Miss Bruce's Grant in Aid of Astronomical Research. A portion of this grant was through Professor E. C. Pickering awarded to Professor Adams for the purchase of an instrument for the Cambridge Observatory; and it was decided to use it in providing a spectroscope for the 25-inch refractor. As the spectroscope is, in this sense, of American origin, it is a special satisfaction to record that the optical parts were (with the exception of the Dallmeyer telephoto combination above referred to) made in America. They were supplied by Mr. Brashear, of Allegheny, and the excellence of their finish and performance is admirable. The mechanical parts of the instrument were made by the Cambridge Scientific Instrument Company; and I gladly take this opportunity of saying how much their care in carrying out the somewhat troublesome design, and the ingenuity with which many difficulties were overcome, have contributed to the success of the instrument.

At the present moment I am not quite prepared to express a final opinion on the success of the general design of the instrument, for there are some points on which I wish to have more precise knowledge. I had hoped to have gained that knowledge before presenting this description, but the weather has been so unfavourable—there has not been a single observing night in the past four weeks—that my hopes have been frustrated, and it has seemed better to present the description at once and leave for a later communication some account of the points I have referred to.

Meanwhile it will probably be of interest to give the following particulars with reference to the spectra photographed with the telephoto camera and with a slit-width of $0^{mm}{\cdot}02$.

With an exposure of 7 minutes, the spectrum of *Venus* comes up with excellent clearness.

With an exposure of 10 minutes, the spectrum of α *Lyræ* from D to H_γ is over-exposed in some parts.

An exposure of 20 minutes gives the spectrum of α *Aurigæ* at H_γ at its best.

An exposure of 30 minutes is enough for γ *Cassiopeiæ*, and with this exposure the doubleness of the bright hydrogen lines, H_β and H_γ is clearly seen.

An exposure of 40 or 50 minutes is required to give a spectrum showing the green bands in α *Orionis* satisfactorily. With this exposure the spectrum is shown from D to λ 4400.

It should, however, be stated that the width of the spectrum is, in the case of the stars, small—rather less than 1 mm. This small

width is found enough for measurement with the microscope, and I have not as yet made any wider spectra for inspection without the microscope.

Expressions for the Elliptic Coordinates of a Moving Point to the Seventh Order of Small Quantities. By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

The mathematical investigations of the Lunar Theory based upon the variations of the elliptic elements have been carried to the seventh order of small quantities. And this requires for completeness the use of the elliptic coordinates to the same order.

But I have failed to find the complete expressions for these coordinates to the seventh order in any of the books in the Observatory library to which I have referred.

I have, therefore, determined these expressions for my own use. And, as they have been found quite independently, they will serve as a verification of any existing results.

The notation adopted is that of Delaunay's "Théorie de la Lune."

The expressions for r, v, V, and U agree identically, to the sixth order of the small quantities, with those given by Delaunay on pages 19 and 55–59, vol. i., of the "Théorie de la Lune," and adopted by him in the formation of the general expression of the disturbing function R. But it appears that Delaunay has subsequently allowed for the terms of the seventh order in the expressions for the elliptic coordinates, with the exception of the terms multiplied by e^7 and γ^7 and the coefficients of periodical terms involving the angle $(7l)$; and he must therefore have been in possession of the complete expressions to the seventh order.

$$
\left\{
\begin{aligned}
\frac{r}{a} = 1 + \frac{1}{2}e^2.& \\
&+\cos l\left(-e+\frac{3}{8}e^3-\frac{5}{192}e^5+\frac{7}{9216}e^7\right).\\
&+\cos 2l\left(-\frac{e^2}{2}+\frac{e^4}{3}-\frac{e^6}{16}\right).\\
&+\cos 3l\left(-\frac{3}{8}e^3+\frac{45}{128}e^5-\frac{567}{5120}e^7\right).\\
&+\cos 4l\left(-\frac{1}{3}e^4+\frac{2}{5}e^6\right).\\
&+\cos 5l\left(-\frac{125}{384}e^5+\frac{4375}{9216}e^7\right).\\
&+\cos 6l\left(-\frac{27}{80}e^6\right).\\
&+\cos 7l\left(-\frac{16807}{46080}e^7\right).
\end{aligned}
\right.
$$